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Hudson Park High School


GRADE 12
MATHEMATICS
June Paper 2


| $\underline{\text { Time }}$ | $: 3$ hours | $\underline{\text { Date }}$ | $:$ June 2019 |  |
| :--- | :--- | :--- | :--- | :--- |
| Examiner | $:$ | SLT | $\underline{\text { Moderator(s) }}$ | $:$ |

## INSTRUCTIONS

1. Illegible work, in the opinion of the marker, will earn zero marks.
2. Number your answers clearly and accurately, exactly as they appear on the question paper.
3. NB - Leave 2 lines open between each of your answers.
4. NB $\quad$ Fill in the details requested on the front of this Question Paper and the Answer Booklet.

- Hand in your submission in the following manner :

Question Paper (on top)
Answer Booklet (below)

- Do not staple your Question Paper and Answer Booklet together.

5. Employ relevant formulae and show all working out. Answers alone may not be awarded full marks.
6. (Non-programmable and non-graphical) Calculators may be used, unless their usage is specifically prohibited.
7. Round off answers to 2 decimal places, where necessary, unless instructed otherwise.
8. If (Euclidean) Geometric statements are made, reasons must be stated appropriately.

## QUESTION 1

1.1. For a certain set of data, the following box-and-whisker diagram was drawn :

1.1.1. Describe the skewness of the data.
1.1.2. What percentage of the data lies between 5 and 6,5 ?
1.1.3. What is the semi-interquartile range of the data?
1.2. The table below shows the number of hours that a Sales Consultant spent with nine clients of his clients, in one year, and the value of their sales (in thousands of rands) for that particular client :

| Number of hours | 30 | 50 | 100 | 120 | 150 | 190 | 220 | 240 | 260 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Value of sales <br> (in thousands of rands) | 270 | 275 | 500 | 420 | 602 | 150 | 800 | 850 | 820 |


1.2.1. Identify the outlier in the data above.
1.2.2. Describe the trend in the data.
1.2.3. Write down the equation of the least squares regression line for this data.
1.2.4. The Sales Consultant forgot to record the sales of one of his clients. If the Sales Consultant spent 80 hours with that client, predict the value of the client's sales, to the nearest thousand rand.
1.2.5. Comment on the strength of the relationship between time spent with a client and the value of their sales. Justify your answer appropriately.
1.2.6. What is the expected increase in sales for each additional hour spent with a client, to the nearest rand ?
1.2.7. Considering the number of hours the Sales Consultant spent with his clients :
(a) Calculate the mean number of hours
(b) Calculate the standard deviation in the number of hours
(c) For how many clients was the time spent with them, by the Sales Consultant, outside of 0,8 standard deviations of the mean?

## QUESTION 2

2. The Ogive curve drawn below, shows the total number of cooldrinks sold at the Tuck Shop during the 30 minute break on a Wednesday :

2.1. Write down the total number of cooldrinks sold by the Tuck Shop during break.
2.2. Determine the modal class of the data.
2.3. How many cooldrinks were bought in the last 10 minutes of break ?
2.4. Use the graph to determine the
2.4.1. lower quartile
(2)
2.4.2. $\quad 60^{\text {th }}$ percentile
for this data. Clearly indicate, on the graph, where any values were read off.

## QUESTION 3

3. ABCD is a parallelogram. $\mathrm{C}(2 ;-1)$ and $\mathrm{D}(-2 ;-5)$. The inclination of line BC is $123,69^{\circ}$. BC crosses the $x$-axis at F .


Calculate the
3.1. gradient of DC
3.2. size of $\alpha$
3.3. size of $\theta$
3.4.1. coordinates of $B$ (to the nearest whole number), and hence
3.4.2. coordinates of A
3.5. length of CD (without the use of a calculator and in simplest surd form)
3.6. area of $\triangle \mathrm{CDK}$, if $\mathrm{BC}=\sqrt{13}$ and K (not shown in the diagram) is a point on BA produced.

## QUESTION 4

4. In the diagram, RQS is a tangent to the circle, with centre T , at the point $\mathrm{Q}(m ; n)$.

The equation of the circle is $x^{2}+4 x+y^{2}-2 y=175$ and the centre of the circle lies on the line QTU whose equation is $2 y-x=4$.

4.1. $\quad$ Calculate the values of $m$ and $n$, showing that $\mathrm{Q}(-14 ;-5)$.
4.2. Determine the equation of the tangent RQS.
4.3.1. Write the equation of the circle with centre T , in the form

$$
\begin{equation*}
(x-a)^{2}+(y-b)^{2}=r^{2} \tag{3}
\end{equation*}
$$

4.3.2. Write down the coordinates of T .
4.3.3. Calculate the coordinates of $U$.
4.3.4. If the circle

- is moved 10 units vertically downwards and 9 units horizontally to the right, and
- has its radius halved
what will its new equation be?


## QUESTION 5

5.1. Given: $\mathrm{A}(-2 ; y)$ and $\mathrm{OA}=\sqrt{13}$

5.1.1. Without the use of a calculator, determine the value of
(a) $y$
(b) $\sin ^{2}\left(\frac{\theta}{2}\right)$ in the form $\frac{a+\sqrt{b}}{c}$ where $a, b, c \in \mathbb{N}$
5.1.2. Calculate $\theta$ correct to two decimal places.
5.2. Solve for :
5.2.1. $-2 \sin 4 x=\sqrt{12} \cos 4 x$
5.2.2. $\quad \sin 2 x+\cos \left(x+30^{\circ}\right)=0$
5.3. Simplify fully, leaving your answer to contain only one trigonometric ratio :

$$
\begin{equation*}
\left(\sin \left(x-1980^{\circ}\right)-\cos (-x)\right)^{2} \tag{5}
\end{equation*}
$$

5.4. Given: $\sum_{x=40^{\circ}}^{50^{\circ}} \sin ^{2} x$
5.4.1. Write $\sin 50^{\circ}$ as a trigonometric ratio of $40^{\circ}$
5.4.2. How many terms are there in the given series ?
5.4.3. Now, without the use of a calculator, evaluate the given series, showing all relevant working out.

## QUESTION 6

6. Given: $f(x)=6 \sin ^{2} x-3$
6.1. On the given set of axes in your Answer Booklet, sketch the graph of $f$ for $x \in\left[-180^{\circ} ; 180^{\circ}\right]$.
6.2. For $f$, state the
6.2.1. period
6.2.2. amplitude
6.3.1. Calculate the general solution of : $f(x)=-1$
6.3.2. Hence, solve for $x$, if $f(x)<-1$ and $x \in\left[-180^{\circ} ; 180^{\circ}\right]$
6.4. If $g(x)=-3 \cos \left(2 x+70^{\circ}\right)+2$ describe the transformation of $f$ to $g$.

## QUESTION 7

7. ABCD is a cyclic quadrilateral. $\mathrm{AB}=3 x, \mathrm{BC}=2 x$ and $\widehat{\mathrm{D}}=90^{\circ}-\theta$ :


Show that: $\mathrm{AC}=x \cdot \sqrt{13+12 \sin \theta}$

## QUESTION 8

8. AD and AE are tangents to the circle DEF. $\mathrm{AC} / / \mathrm{FD}$. Let $\widehat{\mathrm{A}}_{1}=x$.

8.1.1. Prove that $\widehat{\mathrm{E}}_{1}=\widehat{\mathrm{A}}_{1}$
8.1.2. Hence, give the reason why ABDE is a cyclic quadrilateral.

Now, if it is further given that $\mathrm{EF}=\mathrm{FD}$, prove that :
8.2. $\quad \widehat{\mathrm{C}}=\widehat{\mathrm{A}}_{1}$
8.3. $\quad \mathrm{AE}=\mathrm{CD}$
8.4. ABC is a tangent to the circle passing through points $\mathrm{B}, \mathrm{F}$ and D .

## QUESTION 9

9. In $\triangle \mathrm{ABC}, \mathrm{P}$ is the midpoint of $\mathrm{AC} . \mathrm{RS} / / \mathrm{BP}$ and $\frac{\mathrm{AR}}{\mathrm{AB}}=\frac{7}{9}$


Determine :
9.1. $\quad \frac{\mathrm{AS}}{\mathrm{SC}}$
9.2. $\quad \frac{\mathrm{RS}}{\mathrm{BP}}$
9.3. $\frac{\text { area } \triangle \mathrm{SAR}}{\operatorname{area} \triangle \mathrm{ABC}}$

## QUESTION 10

10. Two circles, with centres A and B have radii of R and $r$, respectively.

DX is a tangent to the larger circle at X .
CX is a tangent to the smaller circle at X .


Prove that :
10.1. $\quad X Y^{2}=D Y . Y C$
10.2. $\widehat{\mathrm{A}}_{1}=\widehat{\mathrm{B}}_{1}$
10.3. $\Delta \mathrm{CAY} / / / \Delta \mathrm{YBX}$
10.4. $\quad \frac{r^{2}}{\mathrm{R}^{2}}=\frac{\mathrm{DY}}{\mathrm{CY}}$

